

Smoothed Particle Hydrodynamics -Methodology and Applications

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@NTU

MES, Peking University—History



Peiyuan Zhou

 1952—Established by Peiyuan Zhou at Peking University, 1st Dept. Mech. in PRC

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- **1958**—China's **first** low-speed wind tunnel
- **2000**—State Key Laboratory for Turbulence and Complex Systems
- 2006—Joined in COE
- 2012— Ranked as NO.1 in PRC
- **450** faculty, **3200** B.S., **820** M.S., **280** Ph.D.

The MES serves the world by providing a high-quality learning environment, outstanding research laboratories, and challenging experiences for our students in science, industry and technology.



60th Anniversary Celebration

Fluid Mechanics

Turbulence

•Computational fluid dynamics methods

Aerodynamics

• Propulsion, heat transfer and advanced engines

Solid Mechanics

- •Micro/nano-mechanics, mechanics and physics of complex materials
- •Mechanics of smart materials and structures
- •Elasticity and plasticity
- •Experimental mechanics

Engineering Mechanics

- •Computational mechanics,
- •Fluid-structure interaction
- •Mesh and meshfree methods
- •Structural dynamics
- •Mesh generation

Biomechanics and Medical Engineering

- Cell mechanicsBio-solid mechanicsBio-fluid mechanicsMechanobiology
- •Bio-imaging and bio-instrumentation

Dynamics and Control

- •Cooperative control of multi-agent systems
- •Robust tracking control
- •Redundant control and dynamic control allocation
- •Nonlinear dynamics and control
- Dynamics and control of systems with group symmetries

Research team – Computational FSI

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Computational Mechanics, FSI, Multi-scale modeling

Outline





Background



SPH methodology



SPH for hydrodynamics



SPH for environmental flows



SPH for explosion and impact



Prospects and future directions

1.1 Continuum scale complex problems

- Examples: tsunami, dam collapse, penetration...
- Difficulties:

Free surfaces, deformable boundaries, moving interfaces (for FDM) Large deformations (for FEM)

Complex mesh generation and mesh adaptivity (for both FDM and FEM)



Tsunami with complex interface dynamics



Water discharge and dam collapse with free surface



Explosion and impact

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Difficulties for FDM/FEM



Meshing problems

It is a great challenge to generate or adapt mesh: structured/unstructured, body-fitting, mesh adaptivity, moving mesh, mesh rezoning..., Cost 2/3 work load



Mesh adaptivity



Moving mesh



Mesh rezoning

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Discontinuous problems

Granular flows in environmental, chemical, geophysical, bio-engineering...



Landslide and mud flow



Transport, storage of granular material (corns, chemicals, iron debries etc.)



Traffic, pedestrian flow (evacuation modelling in architectural design)

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1.2 Micro/Nano scale problems

Traditional numerical methods based on continuum scale constitutive equations may not be valid, especially when the dimension diminishes...

1.3 Problems with multi-scale physics Coupling MD with FDM/FEM is complicated and not natural...



Cross section of a bilayer of lipid in water molecules



Evolution of a polymer drop break-up



Water droplets in oil



http://www.worldscientific.com/worl dscibooks/10.1142/5430

Particle Methods for Multi-Scale and Multi-Physics

M B Liu • G R Liu



http://www.worldscientific.com/worl dscibooks/10.1142/9017

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2.1 History

- > **Originally invented** for solving astrophysical problems in open space
- **Recently applied** to general fluid dynamic problems

2.2 Numerical approximation

Weight function (or smoothing function), W, centered on particles and describe continuous or discrete field function,

Kernel approximation:

$$f_i \cong \int f(\mathbf{x}) W_i(\mathbf{x}) d\mathbf{x} \quad f_{i,\alpha} \cong \int f(\mathbf{x}) W_{i,\alpha} d\mathbf{x}$$

Particle approximation:

$$f_i \cong \sum_{j=1}^N f_j W_{ij} m_j / \rho_j \quad f_{i,\alpha} \cong \sum_{j=1}^N f_j W_{i,\alpha} m_j / \rho_j$$

Liu & Liu, WSPC, 2003; 2015



SPH approximations in a twodimensional space

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2.3 SPH equations of motion

Density

Momentum

Energy

Strain rate

$$\begin{split} \rho_{i} &= \sum_{j=1}^{N} m_{j} W_{ij} \qquad \qquad \frac{D\rho_{i}}{Dt} = \sum_{j=1}^{N} m_{j} (\mathbf{v}_{i}^{\beta} - \mathbf{v}_{j}^{\beta}) \cdot \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}} \\ \frac{D\mathbf{v}_{i}^{\alpha}}{Dt} &= -\sum_{j=1}^{N} m_{j} \frac{p_{i} + p_{j}}{\rho_{i} \rho_{j}} \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\alpha}} + \sum_{j=1}^{N} m_{j} \frac{\mu_{i} \varepsilon_{i}^{\alpha\beta} + \mu_{j} \varepsilon_{j}^{\alpha\beta}}{\rho_{i} \rho_{j}} \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}} \\ \frac{D\mathbf{v}_{i}^{\alpha}}{Dt} &= -\sum_{j=1}^{N} m_{j} (\frac{p_{i}}{\rho_{i}^{2}} + \frac{p_{j}}{\rho_{j}^{2}}) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\alpha}} + \sum_{j=1}^{N} m_{j} (\frac{\mu_{i} \varepsilon_{i}^{\alpha\beta}}{\rho_{i}^{2}} + \frac{\mu_{j} \varepsilon_{j}^{\alpha\beta}}{\rho_{j}^{2}}) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\alpha}} \\ \frac{De_{i}}{Dt} &= \frac{1}{2} \sum_{j=1}^{N} m_{j} \frac{p_{i} + p_{j}}{\rho_{i} \rho_{j}} (\mathbf{v}_{i}^{\beta} - \mathbf{v}_{j}^{\beta}) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}} + \frac{\mu_{i}}{2\rho_{i}} \varepsilon_{i}^{\alpha\beta} \varepsilon_{i}^{\alpha\beta} \\ \frac{De_{i}}{Dt} &= \frac{1}{2} \sum_{j=1}^{N} m_{j} (\frac{p_{i}}{\rho_{i}^{2}} + \frac{p_{j}}{\rho_{j}^{2}}) (\mathbf{v}_{i}^{\beta} - \mathbf{v}_{j}^{\beta}) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}} + \frac{\mu_{i}}{2\rho_{i}} \varepsilon_{i}^{\alpha\beta} \varepsilon_{i}^{\alpha\beta} \\ \frac{\partial \omega_{i}}{\partial \mathbf{x}_{i}^{\beta}} &= \sum_{j=1}^{N} m_{j} (\frac{p_{i}}{\rho_{j}^{2}} + \frac{p_{j}}{\rho_{j}^{2}}) (\mathbf{v}_{i}^{\beta} - \mathbf{v}_{j}^{\beta}) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}} - (\frac{2}{3} \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} \mathbf{v}_{ji} \cdot \nabla_{i} W_{ij}) \delta^{\alpha\beta} \end{split}$$

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2.4 Features and advantages

Features

- Particles are used to represent the state of a Particle method system
- Governing equations are discretized and approximated on particles
 - Particles move with the object

of a Particle method s Meshfree method the Lagrangian method SPH combines advantages of FDM & FEM

Merits/demerits

- Mass rigorously conserved, no loss/gain
- Easy to treat large deformations
- Easy to obtain time history of movement and deformations
- Method under development
- Heavier computational cost

2. SPH methodology – Improvements

2.5 Constructing smoothing function

- Criginal SPH empirically specifies that the weight function W satisfies some special requirements like (a) normalization $\int_{\Omega} W(x-x',h)dx'=1$, (b) Delta function property $\lim_{h\to 0} W(x-x',h) = \delta(x-x')$, and (c) symmetric (even) property.
- Weight function constructing conditions

$$f(\mathbf{x}) = \int_{\Omega} f(\mathbf{x}') W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}'$$

$$f(\mathbf{x}) = \sum_{j=0}^{n} \frac{(-1)^{j} f^{(j)}(\mathbf{x})}{j!} (\mathbf{x} - \mathbf{x}')^{j} + r_{n+1}(\mathbf{x} - \mathbf{x}')$$

$$f(\mathbf{x}) = \sum_{j=0}^{n} \frac{f^{(j)}(\mathbf{x})}{j!} \int (\mathbf{x} - \mathbf{x}')^{j} W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' + r_{n+1}(\mathbf{x} - \mathbf{x}')$$

$$M_{0} = \int W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' = 1$$

$$M_{1} = \int (\mathbf{x} - \mathbf{x}') W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' = 0$$

$$M_{2} = \int (\mathbf{x} - \mathbf{x}')^{2} W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' = 0$$

$$M_{n} = \int (\mathbf{x} - \mathbf{x}')^{n} W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' = 0$$

$$W(\mathbf{x} - \mathbf{x}', h) = a_{0} + a_{1}R + a_{2}R^{2} + \dots + a_{n}R^{n}, R = |\mathbf{x} - \mathbf{x}|$$

Liu et al., *JCAM*, 2003

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2.6 Improving accuracy - FPM

The original SPH method does not have C⁰ consistency for particle approximation, so the numerical accuracy of the original SPH method is quite low especially for disordered particles or boundary particles.

Original SPH particle
approximation
$$f_i \cong \sum_{j=1}^N f_j W_{ij} m_j / \rho_j \quad f_{i,\alpha} \cong \sum_{j=1}^N f_j W_{i,\alpha} m_j / \rho_j$$

Finite particle method (FPM), retaining the advantages of SPH with higher order accuracy.

$$\begin{bmatrix} f_i \\ f_{i,\alpha} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^N W_{ij} \,\Delta v_j & \sum_{j=1}^N (\boldsymbol{x}_j^{\alpha} - \boldsymbol{x}_i^{\alpha}) W_{ij} \,\Delta v_j \\ \sum_{j=1}^N W_{ij,\beta} \,\Delta v_j & \sum_{j=1}^N (\boldsymbol{x}_j^{\alpha} - \boldsymbol{x}_i^{\alpha}) W_{ij,\beta} \,\Delta v_j \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^N f_j W_{ij} \,\Delta v_j \\ \sum_{j=1}^N f_j W_{ij,\beta} \,\Delta v_j \end{bmatrix}$$

Numerical scheme of FPM

✓ FPM has C^0 and C^1 consistency for particle approximation.

✓ FPM is not influenced by disordered particle distribution and the selection of smoothing function.

✓ FPM is more computational expensive than the original SPH.

Liu et al., AMM, 2005; ANM, 2006.

2.6 Improving accuracy - FPM



Accuracy and stability of SPH and FPM

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2.7 Adapting SPH (ASPH)



SPH: isotropic weight function



ASPH: anisotropic weight function

✓ different axes to match the particle spacing as it varies in time, space and direction

✓ Capable of catching anisotropic deformation

✓ suitable for problems with large dimension ratio

2. SPH methodology – Improvements

2.7 Adapting SPH (ASPH)



SPH: a metal bar impacting on a solid wall



ASPH: a metal bar impacting on a solid wall



SPH: flow in a micro channel with 800 particles



ASPH: flow in a micro channel with 200 particles

Liu et al., JSW, 2006

2. SPH methodology – Improvements

2.8 Solid boundary treatment (SBT)

- As a meshfree, Lagrangian particle method, SPH is difficult in exactly implementing solid boundary conditions (SBC).
- Virtual particles are usually used to implicitly implement SBC, and the distribution of virtual particles and the calculation of their properties influence the accuracy of SPH simulation.
- A new SBT algorithm—coupled dynamic SBT:
 - 1. Two types of virtual particles: Repulsive and ghost particles
- 2. New repulsive force and new approximation scheme



$$\mathbf{F}_{ij} = 0.01c^2 \bullet \chi \bullet f(\eta) \bullet \frac{\mathbf{A}_{ij}}{r_{ij}^2}$$

$$\chi = 1 - \frac{r_{ij}}{1.5\Delta d} \quad 0 < r_{ij} < 1.5\Delta d$$

pulsive particle

$$\eta = r_{ij} / (0.75h_{ij})$$

ost particle

$$f(\eta) = \begin{cases} 2/3 \quad 0 < \eta \le 2/3 \\ (2\eta - 1.5\eta^2) \quad 2/3 < \eta \le 1 \\ 0.5(2 - \eta)^2 \quad 1 < \eta < 2 \\ 0 \quad otherwise \end{cases}$$

Liu et al., *JHD*, 2013

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2.8 Solid boundary treatment (SBT)



A typical example: dam collapse

With fixed, moving or deformable boundaries

- Dam collapse and water discharge
- Tsunami and flooding
- Wave dynamics
- Liquid drop dynamics









Liu M. B. et al., ArCME, 2010

3.1 Dam collapse and water discharge



Water discharge

Shao, Liu et al., IJCM, 2010

3.2 Water impact – dam break against an obstacle



Liu & Shao, SCPMA, 2012

3.2 Wave impact – water impact on building



Top-bottom view



Front-back view

Initial particle distribution



Without mitigation



Mitigation with a dike

3.3 Injection flow

Frame 001 05 Mar 2011 t= 0.250000E-01 s t= 0.500000E-01 s t= 0.750000 E-01 s t= 0.100000E+00 s t= 0.125000E+00 s t= 0.150000E+00 s t= 0.175	Frame 001 14 Mar 2011 t= 0.000000E+00 s t= 0.250000E-01 s t= 0.500000E-01 s t= 0.750000E-01 s t= 0.100000E+00 s t= 0.125000E+00 s t= 0.150

Water injection into an empty container

Water injection into a filled container

3.3 Water injection













SPH simulation

Water injection

3.4 Multi-phase flows – liquid drop impact on thin liquid film



Ma, Liu et al., *APS*, 2012

3.4 Multi-phase flows – Rayleigh-Taylor instability



Heavier fluid: 1800 kg/m³ Lighter fluid: 1000 kg/m³

Chen, Liu* et al., *JCP*, 2013

3.4 Multi-phase flows – Air bubble rising



$$\rho_{air}/\rho_{water}=0.001 \quad \mu_{air}/\mu_{water}=0.05$$



 $t\sqrt{g/h}$ =2.8, 3.6, 4.0, 4.4, 4.8, 5.2, 5.6, 6.0. The red dots are the Level-Set solution

Chen, Liu* et al., JCP, 2013

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3.5 Water entry – cylinder





Penetration depth

Liu and Shao, ICCEES, 2011

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3.5 Water entry – wedge



Liu et al., IJNMF, 2014

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3.6 Water exit – cylinder



Liu et al., IJNMF, 2014

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5.7E+08

5.4E+08

5.1E+08

4.8E+08

4.5E+08

4.2E+08

3.9E+08

3.6E+08

3.3E+08

2.7E+08

2.4E+08

2.1E+08

1.8E+08

1.5E+08

1.2E+08

9E+07

6E+07

3E+07

0

20

3E+08

6E+08

3.6 Water exit – Projectile launch



Projectile launch directly from water

Projectile launch from a canister

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3.6 Water exit – Projectile launch



Cavity Sections Expansion

Vlandimir et al., artificial supercavitation, 2002.

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3.7 Liquid sloshing



Liquid fuel sloshing in aerospace and aeronautical vehicles.



Water sloshing in a reservoir



LNG sloshing in a LNG ship

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3.7 Liquid sloshing – in a rectangular container



Fluent (up) vs SPH (bottom)



Shao, Liu, and Liu, CAS, 2012

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3.7 Liquid sloshing – in a rotating rectangular container



Yang, Peng and Liu, IJCM 2011

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3.7 Liquid sloshing – Ballast water





Yang, Peng and Liu, IJCM 2011

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3.7 Liquid sloshing – in other shapes of containers







Yang, Peng and Liu, IJCM 2011

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3.7 Liquid sloshing – sloshing mitigation





Vertical baffle



Horizontal ...





Porous ...



Multiple compartmentsShao, Liu et al., EC, 2015.





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3.8 Boom and oil spill — Rigid boom





Yang and Liu, SCPMA 2012

3.8 Boom and oil spill — Flexible boom (SPH+EBG)



Yang & Liu, PRE, 2015; JHD, 2016.

Frontal area and two ends

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3.9 Hydro-elasticity – dam collapse with elastic plate



Shao & Liu, *JHD*, 2013.

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4.2 Landslide

- Fast landslide flow like
- Fluid model non-Newtonian
- Surface topography GIS data



1-Bingham fluid
 2-Pseudoplastic fluid
 3-Newtonian fluid
 4-Dilatant fluid
 5-Ideal fluid

Shear strain rate



Tangjiashan Landslide: Pre- and post-earthquake topographies.

Hu, Liu et al., Environmental Earth Sciences, 2015.

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4.2 Landslide





3D profile SPH model of Tangjiashan landslide

Pre- and post-earthquake topographies

Hu, Liu et al., Environmental Earth Sciences, 2015.

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4.4 Subsurface flows – porous media





SPH simulation

Comparison: SPH (top) and VOF (bottom)

Multiphase flow in a fractured porous media

Liu et al., Water Resources Research, 2007.

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4.4 Subsurface flows – fracture network



SPH VS VOF, Experiment



Multiphase flow in a fracture network





Liu et al., Shock Waves, 2003

Liu et al., Shock Waves, 2013

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5.2 2D confined UNDEX



Pressure field

Bubble evolution

Liu et al., *C&F*, 2003

5.3 Water mitigation (WM)



Water held in plastic bags as a bomb shelter to mitigate blast effects



Contact WM



Peak shock pressure

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5.4 Middle & high velocity impact



Liu et al., Shock Waves, 2006

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5.5 Penetration



Al disk impacting/penetrating al plate at 6180 m/s



Other examples

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5.6 Contact explosion



The shape of the crate is closely related to type, size and shape of the explosive charge

Feng & Liu, Shock Waves, 2013

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5.7 Shape charge with jet formation



Feng & Liu, *C&F*, 2013

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5.7 Shape charge with jet formation & penetration





Feng & Liu, *C*&*F*, 2013

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5.8 Explosive-driven welding



Deribas (1967), Hydrodynamic model

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5.8 Explosive-driven welding





Explosive-driven welding

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Better theories

- ✓ Relationship between particle-particle interactions,
- Theoretical foundation for multi-scale modeling,
- ✓ Fluid-solid interaction model.

Applications:

- Protective technology,
- Coastal engineering and ocean hydrodynamics,
- Civil and environmental engineering,
- ✓ Bio/nano engineering,
- ✓ Movie and animation making...



6. Prospects and future directions

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- Released the first SPH open source code;
- Developing a 3D SPH code for hydrodynamics and environmental flows

1 Features

Code

- 2 and 3D
- Lagrangian, particle method
- Nonlinear
- Explicit algorithm
- Multi-materials, Multi-scales, Multi-physics
- 2 Typical applications
- Free surface flows (breaking waves, dam collapse, flood...)
- FSI (slamming, sloshing, water-ondeck, high speed water entry/exit)
- Extreme load (explosion, impact, penetreation)
- 2 Advantages
- Meshfree: easy in treating large deformation
- Lagrangian particle: easy in treating free surfaces and moving interfaces.
- FSI: simutaneously coupling



Target: 3D SPH code for FSI with wave and currents

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Coupling approaches:

1. Coupling SPH/DEM for problems with solid particles (landslide, e.g.),





landslide



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Coupling approaches:

2. Coupling particle methods with FEM/FDM for boundary enhancement and fluid-solid interaction,









SPH+FEM modeling 911



3. Coupling MD/DPD/SPH for multi-scale simulation,



Multi-scale modeling: from nano to micro and to macro scale 63/68

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Coupling approaches:

4. Coupling multiple physics (viscous, elastic, heat, chemical,





Electrochemical Phenomena: Induced Polarization

• Funding organizations

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DSTA, Singapore

Idaho National Laboratory (INL)

DoE, US

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